Binary Conversion Without Tears

2007 March 28   Anthony Glenn

Doing the conversion of numbers from decimal to binary (and back), using standard methods, involves considerable mental arithmetic. Human beings often struggle with that. The purpose of this document is to explain a more human-friendly method which is easier and less error-prone.

Divide binary numbers into groups of four bits. Each group corresponds to a hex digit. Hex (hexadecimal) is base 16. To convert, use the table below to convert back and forth between hex, binary and decimal, plus do a trivial amount of decimal arithmetic, then you have the answer.

Hex	Binary	Decimal	X16
0	0000	0	0
1	0001	1	16
2	0010	2	32
3	0011	3	48
4	0100	4	64
5	0101	5	80
6	0110	6	96
7	0111	7	112
8	1000	8	128
9	1001	9	144
A	1010	10	160
B	1011	11	176
C	1100	12	192
D	1101	13	208
E	1110	14	224
F	1111	15	240

Table 1

Decimal to Binary

Zero to Fifteen

Simply find the row in the "Decimal" column which corresponds to the decimal number and read the answer in the "Binary" column. After some practice, you will tend to become familiar with the first three columns of Table 1 and you will just remember. If you need more bits, zero fill to the left. The extra insignificant zeros are just padding.

Example: Convert 13 to binary.
Referring to Table 1, decimal 13 means binary 1101. Zero filling, 13 in 8-bit binary would be 0000 1101. The space between the two groups of bits, is an optional notational convenience, so the eye does not get lost when reading longer binary numbers.

Sixteen to 255

Find the largest number in the "X16" column of Table 1, which is less than or equal to the number you are converting (the "input"). Write down the corresponding bits. Subtract the X16 number you

found, from the input; the result should always be between zero and fifteen. Look up the result and write down the bits.

Example: Convert 120 to binary.
Checking the X16 column, 112 is the closest which is still less than or equal. Write 0111 from the binary column. Subtracting, 120 - 112 = 8. Write binary 1000. Final answer: 0111 1000.

256 and Above

It happens that 255 is the largest number which fits into 8 bits. The "dotted decimal" notation for IPv4 addresses is a convenient way of representing a 32-bit number. The 32-bit number is divided into four 8-bit numbers, separated by dots. Then the 8-bit numbers are converted to decimal, still separated by dots. The full 32-bit IP address space is over 4 billion addresses. The advantage of dotted decimal for 32-bit numbers, is that only 8-bit binary conversions are needed, so it is human friendly. Network equipment only sees the 32 bits, not the dots or the decimals.

So, with dotted decimal, we can handle 32-bit binary numbers without needing to know how to do long (pure 32-bit) binary conversions.

Example: Convert 192.12.204.65 to binary.
First convert the 192. The X16 column mentions 192 directly, so write 1100. The subtraction result is zero, so write 0000, then dot. Decimal 12 is easier still, since it is less than 16, the most significant four bits are zero, write 0000. Look up 12, write 1100, then another dot. For 204, 192 is the closest, write 1100. 204 - 192 = 12, write 1100, dot. For 65, see 64, write 0100, 65 - 64 = 1, write 0001. All done. Final answer: 1100 0000.0000 1100.1100 1100.0100 0001

Binary to Decimal

4-bit Numbers

For 4-bit binary numbers, just use Table 1 directly. Look up in the Binary column, write the number from the Decimal column in the same row.

Example: Convert 1011 to decimal.
Look up 1011, write decimal 11. Answer: 11.

8-bit Numbers

Divide the 8-bit number into two 4-bit pieces. Look up the left (most significant) 4 bits in the Binary column, write the decimal number from the **X16** column. Write it in a temporary working place, that number is not the final answer. Look up the right (least significant) 4 bits, write the number from the **Decimal** column. That number is also temporary. Add the two temporary numbers. Since they are decimal, ordinary decimal arithmetic is all that is needed. The sum is the final answer.

Example: Convert 1101 0011 to decimal.
Look up 1101, write 208, from the X16 column. Look up 0011, write 3, from the Decimal column. Add 208 + 3 = 211. Answer: 211.

32-bit Numbers

Assuming a dotted decimal answer is required, then break the 32-bit number into four 8-bit parts, then treat each part as an 8-bit number.

Example: Convert 1111 0011.1001 1011.1111 1011.0000 1111 to dotted decimal.
Look up 1111, temporarily write 240, look up 0011, temporarily write 3, add 240 + 3, then write 243, dot. Similarly, 1001 1011, gives temporary 144 + 11, add, then write 155, dot. Continuing, 1111 1011, gives 240 + 11, write 251, dot. Finally, 0000 1111, gives 0 + 15, write 15.
Answer: 243.155.251.15

Long Binary Numbers

Notice that dotted decimal is just one of many ways a 32-bit number could be represented. It could be written as pure binary, with no dots. The trouble with that is that the eye tends to get lost when reading the number. A 32-bit number could be converted to pure decimal. That would correspond to a decimal number between zero and just over 4 billion. A 32-bit number could be written in hex. Four bits can be converted into one hex digit by inspection, so 32 bits corresponds to 8 hex digits.

Many ways of interpreting 32 bits have been invented. For example, a 32-bit number could be interpreted as four 8-bit ASCII (American Standard Code for Information Interchange) characters. ASCII characters are widely used within IT. The number could be "signed 32-bit", a way of representing integers between -2 billion and +2 billion. There are 32-bit "floating point" formats, a way of representing numbers of widely varying magnitudes, but with limited precision. Be aware that what a particular 32-bit number means, depends on the context.

In Table 1, the Decimal column corresponds to the decimal weight of the right-most digit (least significant digit) of a hex number. The X16 column corresponds to the decimal weight of the next most significant hex digit. Notice that every number in the X16 column is simply the number in the Decimal column, in the same row, multiplied by 16. Hence the name "X16". What we have really been doing, when converting from binary to decimal, is first converting from binary to hex by inspection, then adding up the decimal weights of the hex digits. However, I quietly glossed over the hex bit. To handle longer binary numbers, the table would need to be extended. For example, to handle 16-bit numbers, it would need an "X256" column and an "X4096" column. There are 4 hex digits in a 16-bit number. I have not bothered adding the extra columns.

Human beings are dismayingly ill-suited for long binary conversions. Long binary conversions are only done manually by Really Annoying People, to prove how clever they are. The arithmetic gets a lot harder, errors happen, tears are cried. What have we got computers for? The glorious thing about modern operating systems is that they give you a wonderful calculator for free, which will do binary conversions for you. All you need to do is start it up and put it into **Scientific** mode.

In Windows, click Start, All Programs, Accessories, Calculator. Calculator starts, not looking very impressive. Linux has a similar calculator. Click View, Scientific. Suddenly, there are a whole lot more buttons on the calculator, including some labelled "Decimal", "Hex" and "Binary". To convert from decimal to binary, make sure that Decimal is selected, then enter your decimal number, click Binary and, bingo, there it is in binary. You can even do arithmetic in binary, it's a calculator. Long numbers are no problem at all, it all happens instantly. Similarly, converting from binary to decimal is a snap. Calculator does not understand dotted decimal, but you do, so just do four 8-bit conversions. Long pure binary is easily within your reach, even 64-bit numbers. For really long binary numbers, you might like to manually convert to hex, by inspection, which is easy, then convert between hex and decimal, with the calculator. That way, you do not tend to get so lost in a long binary number.

Calculator is your friend. Enjoy!